Q.P. Code : 60864

**Second Semester M.Sc. Degree Examination, July 2019**

*(CBCS Scheme)*

**Mathematics**

**Paper M 204 T – PARTIAL DIFFERENTIAL EQUATIONS**

*Time : 3 Hours]* *[Max. Marks : 70*

*Instructions :*

1) *Answer any **FIVE** full questions.*
2) *All questions carry equal marks.*

1. (a) Show that $xy + yz + xz = 0$ is the integral surface of first order equation

$$xu_x + yu_y = u$$

with $x^2 + y^2 + z^2 = 4$ and $x + y + z = 2$.

(b) Solve the partial differential equation

$$(1 + x)u_x + yu_y = 0$$

with condition $u(0, y) = e^{-y}$. **(8 + 6)**

2. Solve $p^2 - 3q^2 = u$ with condition $u(x, 0) = x^2$ considering both positive and negative conditions and notations have their usual meaning. **(14)**

3. Obtain the canonical form for elliptic equation from the standard second order linear partial differential equation in two variables and hence determine the canonical form of

$$u_{xx} + x^2 u_{yy} = 0$$ **(14)**

4. (a) Classify and transform the equation

$$y^2 u_{xx} - 2xy\, u_{xy} + x^2 u_{yy} = \frac{y^2}{x} y_x + \frac{x^2}{y} u_y$$ to a canonical form.

(b) Solve :

(i) $$\frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial y^4} - 2\frac{\partial^4 u}{\partial x^2 \partial y^2} = 0$$

(ii) $(D - D' - 1)(D - D' - 2)\, u = e^{2x-y}$. **(7 + 7)**

5. Show that in spherical coordinates $r, \theta, \phi$ defined by $x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$, the Laplace equation takes the form

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2} = 0.$$

(14)

6. Obtain the D'Alembert solution of wave equation $u_{tt} = c^2 u_{xx}, -\infty < x < \infty,\ t \geq 0$ with $u(x, 0) = f(x)$,

$u_t(x, 0) = g(x),\ -\infty < x < \infty$

by reducing it to its canonical form.

(14)

7. (a) Solve $u_t = u_{xx},\ 0 \leq x \leq \pi,\ t \geq 0$

with $u(x, 0) = 5\sin x + 2\sin 5x,\ 0 \leq x \leq \pi$

$u(0, t) = 0 = u(\pi, t),\ t \geq 0$

using method of separation of variables.

(b) Solve $u_t = K u_{xx}, 0 \leq x \leq \infty, t > 0$ with conditions

$u(x, 0) = 0,$

$u_x(0, t) = -u_0, t \geq 0$

using appropriate Fourier transforms.

(7 + 7)

8. (a) Find the solution of heat equation in spherical coordinates.

(b) Determine Green's function for

$u_t = Ku_{xx}, -\infty < x < \infty,\ \ t \geq 0$

with $u(x, 0) = f(x), -\infty < x < \infty$

(7 + 7)